hydraulic jump on a horizontal floor, in which the specific forces before and after the jump are equal and the loss of energy is a consequence of the phenomenon. This will be explained further in the following example. It may be noted at this point, however, that the depths $y_1$ and $y_2'$ shown by the specific-energy curve are the alternate depths; whereas the depths $y_1$ and $y_2$ shown by the specific-force curve are, respectively, the initial depth and the sequent depth of a hydraulic jump.

**Example 3-3.** Derive a relationship between the initial depth and the sequent depth of a hydraulic jump on a horizontal floor in a rectangular channel.

*Solution.* The external forces of friction and the weight effect of water in the hydraulic jump on a horizontal floor are negligible, because the jump takes place in a relatively short distance and the slope angle of the horizontal floor is zero. The specific forces of sections 1 and 2 (Fig. 3-4), respectively, before and after the jump, can therefore be considered equal; that is,

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \qquad (3\text{-}18)$$

For a rectangular channel of width $b$, $Q = V_1A_1 = V_2A_2$, $A_1 = by_1$, $A_2 = by_2$, $\bar{z}_1 = y_1/2$, and $\bar{z}_2 = y_2/2$. Substituting these relations and $\mathbf{F}_1 = V_1/\sqrt{gy_1}$ in the above equation and simplifying,

$$\left(\frac{y_2}{y_1}\right)^3 - (2\mathbf{F}_1^2 + 1)\left(\frac{y_2}{y_1}\right) + 2\mathbf{F}_1^2 = 0 \qquad (3\text{-}20)$$

Factoring,

$$\left[\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2\right]\left(\frac{y_2}{y_1} - 1\right) = 0$$

Then, let

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2 = 0$$

The solution of this quadratic equation is

$$\frac{y_2}{y_1} = \tfrac{1}{2}(\sqrt{1 + 8\mathbf{F}_1^2} - 1) \qquad (3\text{-}21)$$

For a given Froude number $\mathbf{F}_1$ of the approaching flow, the ratio of the sequent depth to the initial depth is given by the above equation.

It should be understood that the momentum principle is used in this solution because the hydraulic jump involves a high amount of internal-energy losses which cannot be evaluated in the energy equation.

The joint use of the specific-energy curve and the specific-force curve helps to determine graphically the energy loss involved in the hydraulic jump for a given approaching flow. For the given approaching depth $y_1$, points $P_1$ and $P_1'$ are located on the specific-force curve and the specific-energy curve, respectively (Fig. 3-4). The point $P_1'$ gives the initial energy content $E_1$. Draw a vertical line, passing through the point $P_1$ and intercepting the upper limb of the specific-force curve at point $P_2$, which gives the sequent depth $y_2$. Then, draw a horizontal line passing through the point $P_2$ and intercepting the specific-energy curve at point $P_2''$, which gives the energy content $E_2$ after the jump. The energy loss in the jump is then equal to $E_1 - E_2$, represented by $\Delta E$.

**3-8. Momentum Principle Applied to Nonprismatic Channels.** The specific force, like the specific energy, varies with the shape of the channel section. In applying the momentum principle to nonprismatic channels, therefore, a three-dimensional plot similar to that shown for the application of the energy principle (Fig. 3-5) can be constructed. For practical purposes, however, this is rarely necessary.



Where there is no intervention of external forces or where these forces are either negligible or given, the momentum principle can be applied to its best advantage to problems, such as the hydraulic jump, that deal with high internal-energy losses that cannot be evaluated if the energy principle alone is used. The following example shows how the momentum principle is applied to the design of a channel transition in which a hydraulic jump is involved.

**Example 3-4.** A rectangular channel 8 ft wide, carrying 100 cfs at a depth of 0.5 ft, is connected by a straight-wall transition to a channel 10 ft wide, flowing at a depth of 4 ft (Fig. 3-10). Determine the flow profile in the transition if the frictional loss through the transition is negligible. If a hydraulic jump occurs in the transition, how can it be eliminated?

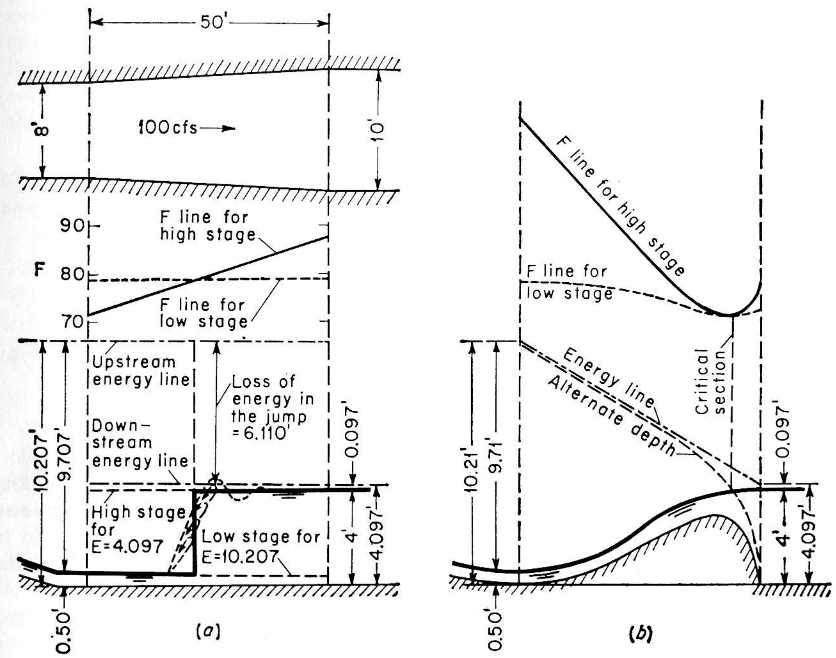


Fig. 3-10. Energy and momentum principles applied to a channel expansion (*a*) with hydraulic jump; (*b*) without hydraulic jump.

*Solution.* From the given data, the total energy with respect to the channel bottom in the approaching flow is $E = 0.5 + [100/(0.5 \times 8)]^2/2g = 10.207$ ft, and in the downstream, $E = 4.0 + [100/(4 \times 10)]^2/2g = 4.097$ ft. It is apparent that this energy difference of 6.110 ft must be dissipated through the transition by some means, since the frictional loss is negligible. Furthermore, the Froude numbers 6.24 and 0.22 of the approaching and downstream flows are, respectively, greater and less than unity, indicating a change of the flow from supercritical to subcritical. Therefore, a hydraulic jump can be expected to occur to dissipate the energy difference and to effect a change in the flow state. Whether this jump will occur within the transition or in the upstream or the downstream channel is, however, to be disclosed by further analysis.

TABLE 3-1. COMPUTATION FOR A CHANNEL EXPANSION DESCRIBED IN EXAMPLE 3-4

| Section width $b$, ft | Low stage $y_1$, ft for $E = 10.207$ | $F_1$ | High stage $y_2$, ft for $E = 4.097$ | $F_2$ |
|---|---|---|---|---|
| 8.00 | 0.500 | 78.6 | 3.940 | 71.9 |
| 8.50 | 0.470 | 78.7 | 3.960 | 75.9 |
| 9.00 | 0.443 | 78.8 | 3.979 | 79.9 |
| 9.50 | 0.419 | 78.8 | 3.987 | 83.6 |
| 10.00 | 0.398 | 78.8 | 4.000 | 87.8 |

Take five sections of the transition with their widths shown in Table 3-1. For the total approaching energy of 10.207 ft, the low stage $y_1$ for each section can be computed by means of Eq. (3-8) or (3-9), or

$$\frac{(100/by_1)^2}{2g} + y_1 = 10.207$$

where $b$ is the width of the section. Similarly, the high stage $y_2$ for a total energy of 4.097 can be computed from

$$\frac{(100/by_2)^2}{2g} + y_2 = 4.097$$

The low- and high-stage lines are then constructed along with the energy lines (Fig. 3-10$a$). After these stage and energy lines are determined, the specific forces $F_1$ and $F_2$ for low and high stages, respectively, at each section are computed and plotted to any convenient scale and datum. The hydraulic jump must occur where the specific forces for the low and high stages are equal, or at the intersection of the $F$ lines. At this section the water surface at low stage will jump to the high stage, as indicated by a vertical line (Fig. 3-10$a$). Actually, however, the jump will take place over a short distance, as shown by the dotted line. The energy loss in the jump is represented by the vertical intercept between the upstream and downstream energy lines, which is equal to 6.110 ft, covering the energy difference between the flows in the connecting channels. By varying the shape of the cross sections of the connecting channels the location of the intersection of the $F$ lines, or the position of the jump, can be altered. Changing the depth of flow in the downstream channel will also change the position of the jump. Generally, an increase in the downstream depth will move the jump upstream, and a decrease in the depth will move the jump downstream.

The hydraulic jump can be eliminated if the energy loss can be dissipated gradually and smoothly. This can be done by introducing proper roughness in the transition, for instance, by bolting cross timbers to the bottom of the transition. It can be assumed in this example that the energy difference of 6.110 ft is dissipated uniformly in the transition by artificial roughness. Thus, the energy line in the transition is simply a straight line joining the total heads of the two end sections (Fig. 3-10$b$). For design purposes, it is convenient first to assume the flow profile and then to proportion the dimensions of the transition so that the jump can be eliminated. In proportioning the transition, the jump is eliminated either by varying the width or by raising the bottom of the transition. In this example, it is assumed that the bottom is to be raised, or "humped" (Fig. 3-10$b$). The subsequent procedure of the computation is to (1) assume the flow profile; (2) compute the velocity head, which is equal to the difference between the total head and the water-surface elevation, at a number of selected sections; (3) compute the velocity and then the water area and depth of flow for each section; (4) determine the elevation of the bottom of the transition, which is equal to the elevation of the water surface minus the depth of flow; (5) compute the alternate depth, since the bottom of the transition is fixed; and (6) compute $F_1$ and $F_2$ lines for the low and high stages, and plot them on a convenient scale. It can be seen that the two $F$ lines intersect and become tangent to each other at a critical section, where the flow changes from low to high stage, that is, from supercritical to subcritical state. If the critical-depth line is plotted, it will intersect the alternate-depth line and the water surface simultaneously at the critical section. Based on the critical-depth line, a line of minimum specific energy can also be constructed. This line should be tangent to the total-energy line at the critical section.



## PROBLEMS

**3-1.** With reference to a channel of small slope and a section shown in Fig. 2-2, ($a$) construct a family of specific-energy curves for $Q = 0, 50, 100, 200, 300$, and 400 cfs, ($b$) draw the locus of the critical-depth point on these curves, ($c$) plot a curve of the critical depth against the discharge, and ($d$) plot a family of curves of alternate depths, $y_1$ vs. $y_2$, for the given discharges.

**3-2.** Construct the specific-energy curve for a 36-in. pipe carrying an open-channel flow of 20 cfs ($a$) on a flat slope, and ($b$) on a 30° slope.

**3-3.** Show that at the critical state of flow the specific-energy head in a rectangular channel is equal to 1.5 times the depth of flow, assuming zero slope and $\alpha = 1$.

**3-4.** Derive the equations for the locus of the critical-depth point on the specific-energy curve and for the curve of critical depth vs. discharge, as obtained in Prob. 3-1.

**3-5.** Prove Eq. (3-12).

**3-6.** Prove Eq. (3-13).

**3-7.** Prove that at the critical state of flow the discharge is a maximum for a given specific energy.[1]

**3-8.** Show that the relation between the alternate depths $y_1$ and $y_2$ in a rectangular channel can be expressed by

$$\frac{2y_1^2y_2^2}{y_1 + y_2} = y_c^3 \tag{3-22}$$

where $y_c$ is the critical depth. Using values of $y_1/y_c$ as ordinates and of $y_2/y_c$ as abscissas, construct a dimensionless graph for the above equation and study its characteristics.

[1] The concept of critical depth based on the theorem of maximum discharge was first introduced by Bélanger [20].