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Discussion

Roll waves simulation using shallow water equations and weighted average flux method

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Discusser:
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The roll waves phenomenon, noticed and described since Maw (1884), has represented an object of interest for researchers and technicians for more than one century. Today it is still drawing the scientists' attention. The numerical simulation developed by the Authors and the following comparison with the experimental results published by R. Brock in the years 1967 and 1969 are a contribution of great interest for the knowledge of the phenomenon. It is a further example of the remarkable results that today it is possible to obtain by proper choice of a specific computational scheme. The simulation method could be a useful instrument for the systematic study of these waves, typical of very steep channel flows.

The writer attended to the roll waves problem many years ago. In an article quoted by the Authors [18] published in Italian in 1961 and translated into English in 1965 (Montuori, 1965), the writer showed that the roll waves appearance or their lack in very steep channels flows could be explained through the high-velocity flow instability theory, if one takes into account the channel length, and he suggested a semi-empirical criterion to predict the roll waves formation. This criterion was deduced by elaborating the results of Vedernikov *et al.* (1947) theory (Powell, 1948), developed in 1947 on the growth of small perturbations in an open-channel flow, and a comparison of the consequent theoretical result with many laboratory and field tests. The writer, thus, showed that the knowledge both of Vedernikov number, $\underline{V} = (2m/p)M\mathrm{Fr}$ (Vedernikov *et al.*, 1947) and of the dimensionless quantity gSx/u_0^2 (u_0 and Fr, velocity and Froude number of an ideal uniform flow; x, distance from the channel inlet; m and p, exponents of a monomial formula of the uniform flow; M, shape factor) allowed to foresee, with enough approximation to the technical applications, if clearly noticeable roll waves could develop in a channel reach. In fact, the position of the experimental points in a diagram $(gSL/u_0^2, \underline{V})$ (L, channel length) clearly showed that the roll waves presence is either connected with the increase of $\underline{V}$ or with the increase of gSL/U_0^2.

Besides, after having drawn a curve representing an approximate relation deduced from a further theoretical development, it was noticed that this curve divided the first quadrant of the diagram into two parts. The points corresponding to flows without roll waves were positioned between the curve and the two coordinate axes. Instead, from the opposite side of the curve, there were all the points corresponding to flows with roll waves, while, in the proximity of the curve, there were also a few points representing some flows where roll waves were not noted. In 1963 (Montuori, 1963) this comparison was extended, with satisfactory results, to further experimental data collected from various sources.

The two dimensionless quantities gSx/u_0^2 and $\underline{V}$ do not take into account neither the boundary layer development near the channel inlet, nor the flow acceleration in the initial reach of the channel. However, they appeared significant to predict the presence of roll waves.

Afterward, in a particular range of cases, they appeared also linked to the roll waves maximum depths.

In fact, in 1984, the writer, considering the previous conclusions, made a further elaboration using the experimental data published by R.R. Brock in 1969 [5]. The results of this elaboration were published only in Italian, in the Proceedings of a seminar organised at Bressanone, Italy (Montuori, 1984). More precisely, the values noticed by Brock were used to calculate the values of the two dimensionless quantities gSx/u_0^2 and $\underline{V}$ pertinent to distances x where the flow maximum depth $h_{\max}$ exceeded of a prefixed rate the uniform flow depth h_0; the data were deduced by the reading of Brock's graphics. Six values of the ratio $h_{\max}/h_0$ were considered, in the range $1, 1 \leq h_{\max}/h_0 \leq 2$. The points obtained in the diagram $(gSx/u_0^2, \underline{V})$ for each value of ratio $h_{\max}/h_0$ were fitted interpolating a specified empirical curve. The diagram which appeared in (Montuori, 1984) is reproduced in Fig. 1m.

Thorsky and Haggman (1970) made a similar elaboration. These Authors used a diagram with the same ordinate $\underline{V}$, but with the abscissa U_0^2/gSx , which was the inverse of the previous one; they only considered the results for which $h_{\max}/h_0 = 1, 1$. The writer made the same use of the abscissa U_0^2/gSx, *Thorsky*'s abscissa, instead of the original abscissa gSx/u_0^2: obviously, even in this way it was possible to interpolate the experimental points by curves with parameters $h_{\max}/h_0$ (Fig. 2m).

Therefore, in Brock's experimental range, the two dimensionless quantities gSx/u_0^2 and $\underline{V}$ are connected together through

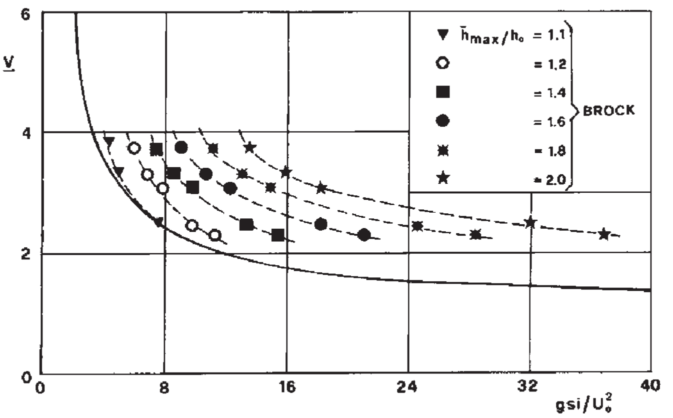

Figure 1m Diagram (gSx/u_0^2, $\underline{V}$) (after Montuori, 1984).

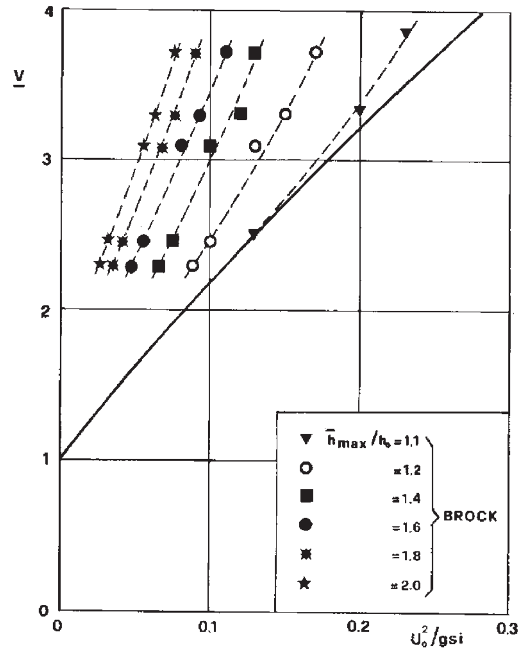

Figure 2m Diagram (u_0^2/gSx, $\underline{V}$) (after Montuori, 1984).

some links only depending on the maximum depth $h'_{max} = h_{max}/h_0$ of roll waves.

The computational method developed by the Authors could be the way to check if similar links, as those found by the writer in 1984, exist outside Brock's experimental range. In this case, the diagram (gSx/u_0^2, $\underline{V}$) could be used to predict the maximum heights of the roll waves in a wider range.

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Reply by the Authors

We thank Prof. Montuori for his interesting discussion and try to answer the points he raises. These points are in synthesis:

- Are Vedernikov number V and the non-dimensional abscissa along the channel gSx/u_0^2 the only independent variables, which all the other non-dimensional variables, as h_{max}/h_0, depend on?
- Can the numerical code extend the range of conditions in which the results presented in Figs 1m and 2m are checked, and has this been done?

The first point requires a dimensional analysis of the case of a stream in a uniform channel potentially developing roll waves as a consequence of instability of the uniform flow.

We can think to impose independently:

- gravity acceleration g;
- water depth and velocity h_0, u_0 of the base uniform flow (typical values of boundary conditions);
- in a given channel friction (and channel) slope would depend on wall roughness and/or viscosity, that we can fix at our choice; as a substitute of these we may assume that slope S is an independent parameter;
- fluid density ρ;
- some perturbation has to be imposed, normally at the boundary, in order to trigger instability, let ε be the very small value of this perturbation relative to the base flow characteristics;
- position along the channel x and running time t are also independent variables.

Let $\{h_0, u_0, \rho\}$ be a dimensional base for these variables. Any non-dimensional dependent variable should depend on the resulting non dimensional independent variables:

$$\left\{gh_0/u_0^2 \equiv \mathrm{Fr}^{-2}, S, \varepsilon, x/h_0, tu_0/h_0\right\} \tag{1}$$

Some reductions are possible: if dependent variables are statistics evaluated over time, the non-dimensional time can be cancelled from the set; if the initial perturbation is constant (practically zero) or is represented by a qualitative variable (rough or smooth inlet, for instance, were used by Brock). The most reduced set of independent variable for a general case is therefore:

$$\{\mathrm{Fr}, S, x/h_0\} \tag{2}$$

This is the scaling rule used by Brock, for instance; in these tests, channel roughness remaining constant, Froude number and slope are related and only one of them can be used.

If our attention is focused on the development of roll waves, we can follow the approach of Liggett (1975) modified by Julien and Hartley (1985, 1986) in order to account for strongly non-uniform velocity distribution over the section.

Unstable waves do propagate downstream with celerity

$$C = \bar{u} + c_0 = \beta_\mathrm{m}\bar{u} + \sqrt{gh + \beta_\mathrm{m}(\beta_\mathrm{m} - 1)\bar{u}^2} \approx \bar{u} + \sqrt{gh} \tag{3}$$

where $\bar{u}$ is the mean velocity over the section (vertical), β_m is the momentum correction factor (the last expression is the asymptotic value for almost uniform velocity distribution)

$$\beta_\mathrm{m} = \frac{1}{\bar{u}^2 A}\int_\mathrm{section} u^2\,\mathrm{d}A \approx 1 \tag{4}$$

c_0 is the celerity of waves relative to fluid mass and is proportional to average velocity times a factor dependent on Froude number and momentum coefficient

$$\frac{c_0}{u} = \beta_\mathrm{m} - 1 + \sqrt{\mathrm{Fr}^{-2} + \beta_\mathrm{m}(\beta_\mathrm{m} - 1)} \approx \mathrm{Fr}^{-1} \tag{5}$$

The rate of increase/decrease of perturbations along the channel is described by Liggett (1975) with the assumption $\beta_\mathrm{m} = 1$ and by Julien and Hartley (1986) relaxing this assumption. The perturbation of the free surface elevation satisfy equation

$$\frac{\partial^2 h'}{\partial\xi\partial\eta} - \beta\left(\frac{\partial h'}{\partial\eta}\right)^2 + \gamma\frac{\partial h'}{\partial\eta} = 0 \tag{6}$$

where $\xi \equiv x$ is the usual longitudinal coordinate and η is a longitudinal coordinate relative to an origin moving with the wave, whose solution is

$$\frac{\partial h'}{\partial\eta} = \frac{\varepsilon}{\beta\varepsilon/\gamma + \exp(\gamma\xi)} \tag{7}$$

The solution shows that the steepness of the perturbation increases along the channel if γ is negative starting from the initial value ε the length scale of this rate of increase is for rectangular channel and laminar flow

$$\gamma = \frac{S}{h}\mathrm{Fr}^{-2}\left(1 - 2\frac{c_0}{\bar{u}}\mathrm{Fr}^2\right)\left[2 + \frac{c_0}{\bar{u}} + \left(\frac{c_0}{\bar{u}}\right)^{-1}\mathrm{Fr}^{-2}\right]^{-1} \tag{8}$$

For a different flow regime (Liggett, 1975) or a different channel only the numerical coefficients in the above formula do change.

Unstable conditions are characterized by negative values of the term in round brackets.

These equations shows that, as far as the evolution of perturbations is concerned, if Fr is one of the parameters, the longitudinal co-ordinate $\xi \equiv x$, stream depth h and slope S appears only through the combination xS/h, answering from a theoretical point of view to the first point raised.

Actually this argument fails quite early, since as soon as the denominator of Eq. (7) vanishes a bore is formed and original equation should be adapted. If this remains true above this limit cannot be proved experimentally only from Brock results, because roughness was not varied in these experiments.

A possible explanation is the following. If we assume that propagation can be described by shallow water equations, that for the sake of simplicity we write in the mass and head balance form (or momentum balance form) for a prismatic channel of constant slope angle θ

$$\begin{aligned} &\frac{\partial A}{\partial t} + \frac{\partial}{\partial x}(\bar{u}A) = 0 \\ &\frac{1}{g}\frac{\partial\bar{u}}{\partial t} + \frac{\partial}{\partial x}\left(h\cos\theta + \frac{\bar{u}^2}{2g}\right) + \frac{\tau_\mathrm{b}}{\rho g R} - \sin\theta = 0 \end{aligned} \tag{9}$$

that can be written in the form

$$\frac{\partial}{\partial t}(\rho A\bar{u}) + \frac{\partial}{\partial x}(\rho g\cos\theta\cdot h_\mathrm{G}A + \rho\bar{u}^2 A) + \bar{\tau}_\mathrm{b}P - \rho g\sin\theta A = 0$$

First we can rescale

- x and t into $x' = x\sin\theta/h_0$ and $t' = tu_0\sin\theta/h_0$,
- A and $\bar{u}$ into $A' = A/A_0$ and $u' = \bar{u}/u_0$,

where subscript '0' denotes values for a stationary and uniform base flow.

Then, we cancel in the resulting equations the constant factors, including $\sin\theta$ (or $\rho g\sin\theta A$) appearing in all but one term.

Finally, we assume that the resistance relation has the monomial expression

$$\frac{\tau_\mathrm{b}}{\tau_\mathrm{bo}} = \left(\frac{\bar{u}}{u_0}\right)^n\left(\frac{R}{R_0}\right)^{-m}$$

so Eq. (9) can be written as

$$\begin{cases} \dfrac{\partial A'}{\partial t'} + \dfrac{\partial}{\partial x'}(u'A') = 0 \\ \mathrm{Fr}_0^2\dfrac{\partial u'}{\partial t'} + \dfrac{\partial}{\partial x'}\left(h'\cos\theta + \mathrm{Fr}_0^2\dfrac{u'^2}{2}\right) \\ \quad + u'^n R'^{-(m+1)} - 1 = 0 \end{cases} \tag{10}$$

or also in the form

$$\mathrm{Fr}_0^2\frac{\partial}{\partial t'}(u'A') + \frac{\partial}{\partial x'}(\cos\theta\cdot h'_\mathrm{G}A' + u'^2 A') + A'(u'^n R'^{-(m+1)} - 1) = 0$$

The solution of this system (10) depends only on parameters Fr_0, n, m and $\cos\theta$, i.e. if slope is not so great that $\cos\theta \cong 1$ and flow is everywhere either laminar or turbulent (n and m constant for the case) the scaled solution depends only on Fr_0 and scaled independent variable x' and t', beside on scaled boundary condition.

Our numerical simulations of Brock's (1970) experiments, based on shallow water equations, satisfy this condition as it is shown in Fig. 1 which represent the crest to normal depth ratio h_max/hn along the channel length x adimensionalized by normal flow depth and channel slope S.

We repeated the simulations for the cases of Fr = 3.45, 3.71 increasing channel bed roughness (from 10^{-5} to 10^{-4} m) and slope (from 5 to 10%) and maintaining Brock's flow depth of the

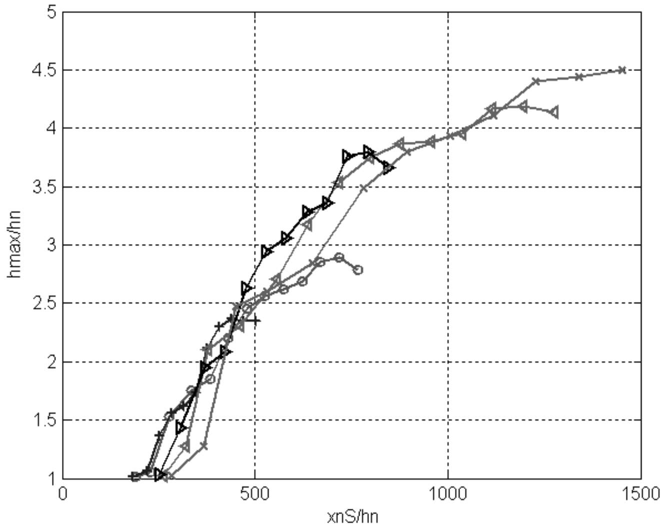

Figure 1 Increase rate of relative crest height $h_{\max}/hn$ versus xS/hn for the simulations of Brock's tests reported in Zanuttigh and Lamberti (2002).

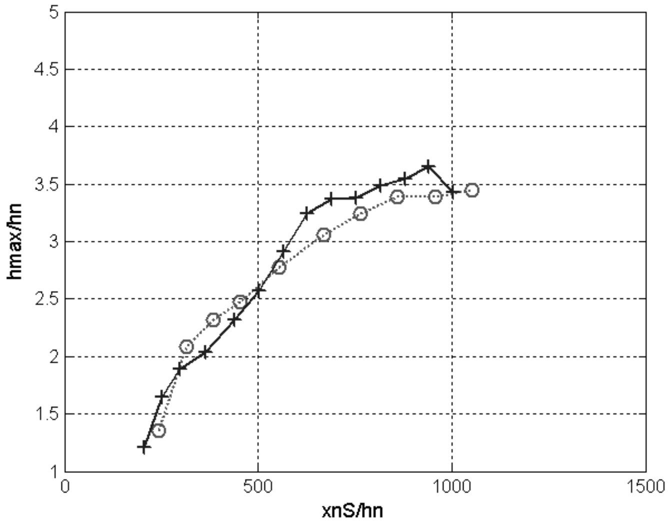

Figure 2 Increase rate of relative crest height for Fr = 3.45, 3.71; roughness 10^{-4}, slope 10%.

base uniform flow. Results are shown in Fig. 2: wave crests reach higher values and wave overtaking phase begin more downstream the channel inlet than in the corresponding cases reported with the same symbols in Fig. 1.

Based on Figs 1 and 2 we can therefore conclude that the first point as presented by us is theoretically proven if channel steepness is moderate and shallow water conditions are satisfied, whereas further simulations confirm the proof numerically.

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